Exercise 1.

Consider the function f defined by $f(x) = |\ln(x-2)| + (x-4)$ and (C_f) its representative curve in an orthonormal frame $(O, \boldsymbol{i}, \boldsymbol{j})$.

1. Find the domain of definition of f.
2. Calculate $\lim_{x \to 2^+} f(x)$ and determine an asymptote at (C_f).
3. Calculate $\lim_{x \to +\infty} f(x)$ and $f(3)$.
4. Determine the expression of the function $g_1(x) = f(x)$ for $2 < x \leq 3$ then the expression of the function $g_2(x) = f(x)$ for $x \geq 3$.
5. Determine ${g_1}'(x)$.
6. We denote by $\left(C_{g_1}\right)$ the curve representative of $g_1(x)$ in $(O, \boldsymbol{i}, \boldsymbol{j})$. Find the equation of the tangent (T_1) to (C_{g_1}) at the abscissa point $x = 3$.
7. Determine ${g_2}'(x)$.
8. We denote by $\left(C_{g_2}\right)$ the curve representative of $g_2(x)$ in $(O, \boldsymbol{i}, \boldsymbol{j})$. Find the equation of the tangent (T_2) to (C_{g_2}) at the abscissa point $x = 3$.
9. Compare $g_1'(3)$ and ${g_2}'(3)$. Deduce if f is differentiable at $x = 3$.
10. Draw up the variation table of g_1.
11. Show that the equation $g_1(x) = 0$ admits a unique solution α_1 and verify that $2.1 < \alpha_1 < 2.2$.
12. Draw up the variation table of g_2.
13. Show that the equation $g_2(x) = 0$ admits a unique solution α_2 and verify that $3.5 < \alpha_2 < 3.6$.
14. In order to deduce the graph of (C_f), draw on the same graph (C_{g_1}) and (T_1) then (C_{g_2}) and (T_2).

Exercise 2.

A player has a balanced die whose sides are numbered from 1 to 6, and three urns, U_1, U_2 and U_3 each containing n balls, where n denotes a natural number greater than or equal to 3. We note that the urn U_1 contains 3 black balls, urn U_2 contains 2 black balls and urn U_3 contains 1 black ball.

When the player rolls the dice:

- If he gets the number 1, he randomly takes a ball from the urn U_1, notes its color, and puts it back in U_1.
- If he gets the number 3 or 6, he randomly takes a ball from the urn U_2, notes its color and puts it back in U_2.
- If the number brought up by the die is neither 1, nor 3, nor 6, he randomly takes a ball from the urn U_3, notes its color and puts it back in U_3.

We denote by A, B, C and N the following events:

- A: "The die brings number 1".
- B: "The die brings 3 or 6".
- C: "The die brings up a number which is neither 1, nor 3 and nor 6".
- N: "The drawn ball is black".

1. The player plays a game.
    a. Calculate $P(N|C)$.
    b. Show that the probability that he obtains a black ball is equal to $\frac{5}{3n}$.
    c. Calculate the probability that the dice brought 1 giving that the drawn ball is black.
    d. Determine n so that the probability of obtaining a white ball is greater than ½.
    e. Determine n so that the probability of obtaining a black ball is equal to $\frac{1}{30}$.
2. In this question, n is chosen such that the probability of getting a black ball while playing a game is equal to $\frac{1}{30}$. The player plays 2 games, independent of each other. Calculate the probability that he will get a black ball at least once.