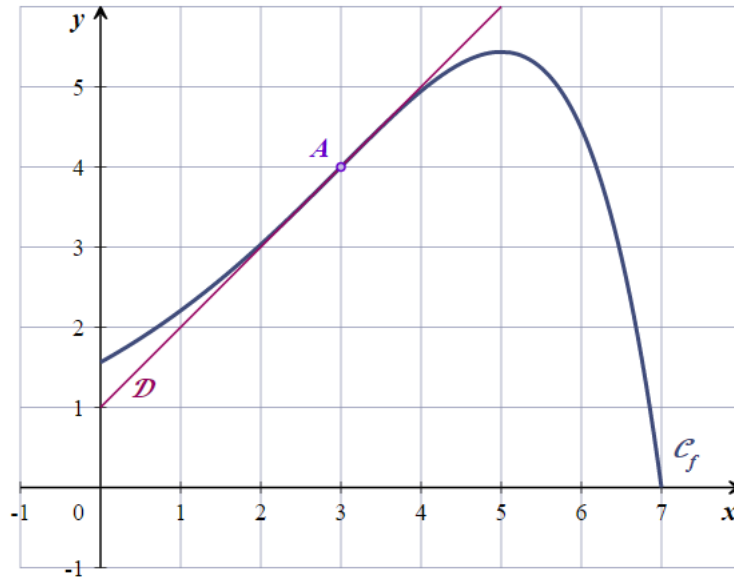


**Problem 1. (60 pts)**

Let  $f$  be a function defined over the interval  $[0; 7]$  by  $f(x) = (ax + b)e^{0.5x-1.5}$ , where  $a$  and  $b$  are two real numbers. We admit that the function  $f$  is twice differentiable. We denote by  $f'$  its derivative and  $f''$  its second derivative.

The representative curve  $C_f$  of the function  $f$  is given below in an orthonormal coordinate system.

Line  $\mathcal{D}$  is tangent to curve  $C_f$  at point  $A$ .



1. By graphical reading, give the values of  $f(3)$  and  $f'(3)$ .
  2. Show that for any real  $x$  of the interval  $[0; 7]$ , we have  $f'(x) = (0.5ax + a + 0.5b)e^{0.5x-1.5}$ .
  3. (a) Deduce from the two previous questions, by solving a system, that  $a = -1$  and  $b = 7$ .  
(b) Give the expressions of  $f(x)$  and of  $f'(x)$ .
  4. (a) Study the sign of  $f'(x)$  over the interval  $[0; 7]$ .  
(b) Deduce the table of variations of the function  $f$  over this same interval.
  5. Show that in the interval  $[5; 7]$ , the equation  $f(x) = 4$  admits a second solution  $\alpha$ .
  6. Definitions (convexity of a function).
    - We say that a function  $f$  is convex over an interval  $I$  if  $f''(x) \geq 0 \forall x \in I$ .
    - We say that a function  $f$  is concave over an interval  $I$  if  $f''(x) \leq 0 \forall x \in I$ .
- (a) Show that  $f''(x) = \frac{(3-x)e^{\frac{x-3}{2}}}{4}$ .
- (b) Study the convexity of the function  $f$  over the interval  $[0; 7]$ .

مباراة الدخول للعام الجامعي 2021-2022

مدة الامتحان: ساعة ونصف

مسابقة في الثقافة المتخصصة / اختصاص علم البيانات

**Problem 2. (20 pts)**

1. After determining the domain of definition, solve the equation

$$\ln(2x + 1) - \ln(x + 3) = 1.$$

2. Solve in  $\mathbb{R}$  the equation

$$\frac{e^x - e^{-x}}{e^{2x} + e^x} = -2$$

**Problem 3. (20 pts)**

A 9-key keypad allows you to dial the entry code of a building, using a letter followed by a 3-digit number, knowing that repetition of digits is authorized.

|   |   |   |
|---|---|---|
| 1 | 2 | 3 |
| 4 | 5 | 6 |
| A | B | C |

1. How many different codes can we form?
2. How many codes are there without the number 1?
3. How many codes have the number 1 at least once?
4. How many codes are there with distinct digits (without repetition)?