الجامعة اللبنانية- كلية الاعلام
مباراة الدخول للعام الجامعي 2022-2023
مسابقة في الرياضيات / اختصاص علم البيانات / مدة الامتحان: ساعة ونصف

Problem I. (25 points)

The evolution of a turnover of a company for the period 2001 -2010 is given in the table below:

Year	2001	2003	2006	2008	2010
Rank of the year x_i	1	3	6	8	10
Turnover expressed in billions of dollars y_i	64.8	68.7	72.7	77.1	82.1

1. Calculate the means $\bar{x}$ and $\bar{y}$.
2. Represent, in an orthonormal system, the scatter plot of the points associated to this series as well as the mean point G.
3. Determine, using the least squares method the equation of the regression line ($D_{X/Y}$) of Y onto X then draw the line in the same system.
4. Estimate the turnover in the year 2012.
5. Starting from which year the turnover of this company exceeds 100 billion of dollars.
6. Calculate the percentage of the increase of the turnover from 2006 to 2010.

Problem II. (The questions in this problem are independent) (30 points)

1. What is the simplest value of $A = 2\ln(6) - 3\ln(10) + \ln(125) + 2\ln(2.5)$?
2. What is the domain of definition of the function $(x) = \frac{\ln(x-1)}{5-x^2-4x}$?
3. What is the solution of the equation $\ln(3-2x) - \ln(x) = 2 + \ln(3)$?
4. What is the derivative of the function $f(x) = xe^{2x} + \frac{\ln(x+1)}{3x-5}$?
5. What is the limit of the function $f(x) = \frac{3+2\ln(x)}{1+\ln(x)}$ at $+\infty$?

Problem III. (45 points)

Consider the function f defined over $\mathbb{R}$ by $f(x) = \frac{9}{4+e^x}$ and denote by (C) its representative curve in an orthonormal system of origin O.

1) Find the domain of definition of $f(x)$ and use it to deduce the asymptotes.
2) Verify that $f(x)$ is decreasing and set up its table of variations.
3) Determine the coordinates of the point of intersection of (C) with the y-axis.

4) Use the informations found in parts a) b) and c) to draw the curve of the function $f(x)$.

5) Definitions (convexity of a function).

- We say that a function f is convex over an interval I if $f''(x) \geq 0 \ \forall x \in I$.
- We say that a function f is concave over an interval I if $f''(x) \leq 0 \ \forall x \in I$.

a) Prove that $f''(x) = \frac{9(e^x-4)e^x}{(e^x+4)^3}$.

b) Study the convexity of the function $f(x)$.