Lebanese University

Faculty of Information

Entrance Exam / 2023-2024

Maths Exam – duration 1h30

__Answer the following questions:__

Problem 1 (50 points):

Part A.

Let f be the function defined on $]0;\ +\infty[$ by $f(x) = x + \ln\left(\frac{x}{2x+1}\right)$.

We denote by (C) the representative curve of f in an orthonormal system $(O; \vec{i};\ \vec{j})$.

1) Determine $\lim\limits_{x \to 0} f(x)$ and deduce an asymptote at (C).
2) Determine $\lim\limits_{x \to +\infty} f(x)$.
3) Study the variations of f and set up the table of variations of f.
4) Show that the line (d) with equation $y = x - \ln(2)$ is asymptotic to (C) and study the position of (C) with respect to (d).
5) Draw (C).
6) Show that the equation $f(x) = 0$ admits on $]0\ ;\ +\infty[$ a unique solution α such that $1 < \alpha < \frac{5}{4}$

Part B.

Let g be the function defined on $]0;\ +\infty[$ by $g(x) = (2x + 1)e^{-x}$.

We denote by (γ) the representative curve of g in an orthonormal system $(O; \vec{i};\ \vec{j})$.

1) Determine $\lim\limits_{x \to +\infty} g(x)$ and deduce an asymptote at (γ).
2) Study the variations of g and set up the table of variations of g.
3) Draw (γ).
4) Show that α is solution of the equation $g(x) = x$
5) Show that if $x \in \left[1; \frac{5}{4}\right]$, then $g(x) \in \left[1; \frac{5}{4}\right]$
6) Study the variations of g' and show that for all $x \in \left[1; \frac{5}{4}\right]$ then $|g'(x)| \leq \frac{1}{2}$

Problem 2 (25 points):

An urn U_1 contains three white balls and two black balls.

An urn U_2 contains a white ball and a black ball.

We draw at random and simultaneously two balls from U_1 and one ball from U_2 and we thus obtain three balls, the draws in each urn being equally probable.

1) We denote by E the following event:
   « Among the three balls drawn there are exactly two white balls »
   Show that $P(E) = \frac{9}{20}$.
2) Let X be the random variable which associates the number of white balls obtained with each draw.
   a. Determine the possible values.
   b. Determine the probability law of X
3) Calculate the probability of having drawn one and only one white ball from urn U_1 knowing that we have drawn two white balls.

Problem 3 (25 points):

The table below gives the consumption in liters of pure alcohol per inhabitant aged 15 and over, on French territory between 1998 and 2004.

Year	1998	1999	2000	2001	2002	2003	2004
Quantities	14.6	14.4	14	14.2	13.9	13.4	13.1

1) Calculate the rate of change in consumption between 2000 and 2001.
2) On a sheet of graph paper, construct the cloud of points of this statistical series in an orthogonal coordinate system. We will take for graphic units: 1 cm for a year on the abscissa, 2 cm for 1 L of pure alcohol on the ordinate. The axes will be graduated from 1996 on the abscissa, and from 10 on the ordinate.
3) Determine the coordinates of the mean point G.
4) We consider points A and B with respective coordinates (1999; 14,4) and (2003; 13,4).
   a) Draw the line (AB).
   b) Determine an equation of the line (AB) by rounding the leading coefficient and the intercept to the hundredths.
   c) Is point G on this line?
5) Considering the shape of the cloud of points, we estimate that the evolution of the quantity of pure alcohol consumed is modeled until 2010 by the affine function of which the line (AB) is a graphic representation.
   a) What consumption can we then predict for 2008?
   b) The objective for 2008 is to obtain a 20% reduction compared to the quantity absorbed in 1998. With this adjustment, can the objective be achieved?